Computer Graphic Simulation of Light Reflection from Paper

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Abstract

Computer graphic reflection modeling techniques were used to study the surface reflection of paper. Models of the subsurface microstructure of paper were constructed and the reflection of light from these models was simulated using Monte Carlo methods. Measurements of the spatial distribution of light reflected from several paper samples were also taken. Factors studied that affect reflection from paper include formation, fiber orientation, and coating. Good agreement was found between the simulations and both the analytical and measured reflectances for the paper samples.

Introduction

Computer graphic algorithms for image synthesis make it possible to create synthetic pictures of real world scenes. Mathematical models of three dimensional objects are constructed, the behaviour of light in the environment is simulated, and a picture of the scene is made from a particular vantage point. The sophistication of computer graphic reflection models has increased to the point that it is now possible to accurately simulate the appearance of a material. This opens the possibility for image scientists of using computer graphics to study the reflection of light from the printed page.

This article takes one step down this path by investigating the use of realistic image synthesis techniques to model the reflection of light from paper. To accomplish this we employ a tool called a *virtual* goniospectrophotometer that we have developed and documented elsewhere (Gondek, Meyer, and Newman, 1994; Schramm, Gondek, and Meyer, 1997). This tool allows us to build models of surface microstructure, to simulate how light interacts with this surface model, and to make a picture of an object with the modelled surface reflection properties. We also make comparisons between our simulations and the analytical and measured reflectance properties of paper.

This article is divided into four additional major sections. In the second section we review computer graphic reflection modeling techniques. In the third section we discuss analytic techniques that have been developed for determining paper reflection. In the fourth section we describe our measurements and simulations of the reflection of light from paper. In the final section we summarise the paper and draw some conclusions.

Computer Graphics Reflection Modeling

Realistic synthetic images are generated by simulating the interaction of light with the objects modelled in a scene. An image is constructed by determining the light scattering events that take place in the scene and then calculating the characteristics of the light that reaches the eye of an observer or synthetic camera placed in the environment. A global illumination model, such as raytracing, is used to describe the way light scatters from surface to surface. Local illumination models describe the specific characteristics of the light scattered from a point on a given surface. A primary focus of research in the field of realistic image synthesis has been the determination of appropriate local reflectance functions for a large variety of materials. A software tool recently developed by Gondek, Meyer, and Newman (1994) allows for the determination of general surface reflectance functions based on computer simulations of the interaction of light with morphological models of the microstructure of these surfaces. This software tool simulates the operation of goniospectrophotometer, which is a measurement instrument used to record the complete spectral and spatial characteristics of light scattered from a surface.

In computer graphics, reflectance functions are described by various local illumination models. These models are designed to calculate the appearance of a object given the spatial orientation and spectral intensity of the light incident upon its surfaces. Some of the phenomenon light may undergo when it is incident upon a object include: reflection from the object's surfaces, transmission through the object, and absorption by the object. When light interacts with complex surfaces, such as the ones we will be studying in this work, it may be subject to all three phenomena. These three mechanisms contribute to the overall scattering characteristics of a material. The spectral characteristics of this scattering define such attributes as the color or hue of an object whereas the spatial characteristics of this scattering define the gloss or luster of the object. By capturing these effects, a local reflectance function can be thought of as a mathematical description of these optical characteristics. Given the spectral and spatial properties of the incident

illumination, a local reflectance function will describe the spectral and spatial distribution of the light scattered from a given surface.

In order to accurately represent the scattering of light from a surface, a reflectance function must relate the ratio of incoming light to exiting light for all angles both in the incidence and exitance directions. In addition, these ratios should be separately available across the entire spectrum of visible light in order to accurately represent the reflectance properties of surfaces whose spatial and specular characteristics are intertwined. The specular composition of light scattered from surfaces such as thin films, for example, is directly related to the spatial geometry of the illumination and viewing conditions. In order to capture the effects from such a surface, the reflectance function must be able to represent the spectral composition of the scattered light for all directions relative to the surface normal. The bidirectional reflectance distribution function (BRDF) introduced to the computer graphics community by Kajia (1986) is such a representation. The BRDF description specifies the ratio of reflected radiance to incident flux density for all incident and reflected directions. In the most general form of the BRDF, this ratio is separately recorded for all wavelengths across the spectrum.

Kajia showed that previously developed local illumination models could be recast as special cases of the BRDF. He also used the BRDF representation to capture anisotropic reflectance functions, which are the result of preferential orientation in the microgeometry of a surface. Materials such as cloth, or brushed metal, have microfeatures that are preferentially oriented in the plane of their surfaces and thus will produce reflectance functions which are not symmetric about the normal of these surfaces. This anisotropy in the reflectance function can only be represented by a bidirectional function such as the BRDF. Through this work, Kajia was able to broaden the class of surfaces that could be accurately represented by reflectance functions used in computer graphics.

Several computer graphics researchers have generated BRDF's by first creating an explicit surface model and then simulating how light reflects from this surface. Cabral et. al. (1987) modelled the surface using an array of triangular microfacets. The light scattered from the surface during the simulation was captured in an equally subdivided hemisphere which was then fit with a set of spherical harmonics in order to represent the BRDF. Westin et al. (1992) extended the work of Cabral with a more elaborate surface representation which involved directly computing the spherical harmonic function during a ray casting simulation. Westin used this new approach to produce realistic reflectance functions for anisotropic surfaces such as brushed aluminum and textiles. Recently Lalande and Fournier (1997) have used wavelets to represent the BRDF that is created by a light reflection simulation.

The spherical harmonic and wavelet representations of the BRDF used in these previous studies is useful in characterizing the entire spatial distribution of the light scattered from a surface. However, it does not contain a provision for the spectral component of this scattering. A separate spherical harmonic function must be used for each wavelength in order to capture the spectral aspect of the scattering from a surface. This hinders the efficiency of the representation and complicates its calculation. A BRDF representation presented by Gondek, Meyer, and Newman (1994) addresses this issue. This representation integrated the spectral and spatial characteristics of the BRDF in a single data structure which was then used in a Monte Carlo raytracer to generate realistic images of a broad variety of complex surfaces. The reflectance functions for these surfaces were precomputed by simulating the interaction of light with accurate geometric models of the surfaces. These simulations incorporated both the surface reflectance, and subsurface scattering from the materials in question. This latter aspect of the interaction of light with a material had not been addressed by either previous method. The software developed for these simulations represents a virtual goniospectrophotometer. An actual goniospectrophotometer is a device which is used to measure the spectral and spatial scattering of light from a surface.

The method used in the Gondek, Meyer, and Newman virtual goniospectrophotometer employed in our research is similar to the one used by Westin to generate reflectance functions for explicitly modeled surfaces. Numerous light rays are shot into a model from a given incident direction, and their scattering is captured in order to calculate the reflectance function for the model. The modeled surface can be composed of various semi-opaque geometric primitives and tessellated height fields. Wavelength dependent indices of refraction and absorption can also be specified for these objects, with the goal being to model the actual structure under consideration as accurately as possible. The rays shot at these models contained polarization and phase information and could represent either a single wavelength or an array of wavelengths along the visible spectrum. A technique similar to raytracing was used to track each ray as it interacted with the structures in the surface model and a variety of optical effects were captured this way. The behavior of a ray at any given optical interface is governed by the wavelength dependent indices of refraction at the interface and the polarization state of the ray. Depending on user defined parameters, a statistical approach can also be used to control ray branching at an interface. Absorption is accounted for by specifying wavelength dependent absorption coefficients for the elements of the model and then using these to calculate ray attenuation according to the Bouger-Beer laws. Interference effects were also accounted for by keeping track of phase information as rays were transported through the model. Rays that exited the surface model in parallel (within a small tolerance) were summed according to the Fresnel-Arago laws, as described in Gondek (1994). This allowed the effects of thin film interference to be captured. The software could also be run in a mode that detected the existence of multiple thin films and analytically calculated the reflectance for these structures. This approach avoided



Figure 1: The measured reflectance functions for the Mountie Matte sample, both inked (light) and non-inked (bold). a) 0 deg incident illumination. b) 20 deg incident illumination..

the exponential ray branching that would occur if conventional techniques were used.

The scattering that results from the interactions of these multiple rays with the surface model is captured by a novel data structure which is well tailored for this type of simulation. A geodesic sphere is used to represent the exiting directions above and below the surface, with each facet of this sphere designed to capture the exiting rays over a discrete range of possible exit directions. The sphere dynamically subdivides during the course of the simulation so that regions containing high degrees of variation are more finely discretized than more uniform regions. The rays captured in each facet of the dome are then used to calculate and store the spectral reflectance ratio with respect to the incident direction. This ratio represents the spectral reflectance for the range of exit angles that the facet covers. Spectral transmitance ratios are similarly created by capturing the rays exiting the bottom of the model. Since the work presented in this paper relies only on the reflectance ratios, references to the transmittances are left out of the following discussions.

The data structure as described is adequate enough to represent the spectral reflectance ratios for one incident angle across all exitance directions; however, in order to represent the full BRDF, this simulation must be re-run for a number of incident directions. Rather than creating a separate data structure for each incident angle sampled, the same data structure is reused and further elaborated by



Figure 1: The measured reflectance functions for the Mountie Matte sample, both inked (light) and non-inked (bold). c) 45 deg incident illumination. d) 75 deg incident illumination.

simulating subsequent sets of rays at other incident directions. The spectral reflectance ratios recorded in the structure are tagged in order to keep track of which incident direction they belong to. Using this single unified data structure has the drawback that the area of highest variance, which in turn causes the greatest degree of elaboration in the structure, is in the specular direction. For surfaces that exhibit significant specular reflection, the structure is very inefficient since it is highly elaborated across the entire structure when multiple incident directions are sampled. In order to avoid this inefficiency, the entire data structure is rotated for each incident angle such that the specular direction is coincident for all the incident angles sampled. This effectively concentrates the portion of the data structure that has to be highly subdivided, thereby saving significant amounts of space at run time. This technique also eases the interpolation necessary during rendering in order to produce a smooth reflectance function from the discretely sampled incident directions.

Each simulation run thus produces the spectral reflectance ratios for a family of discrete incident directions. A set of ratios for a given arbitrary incident angle is then generated by interpolating the ratios across the four nearest discrete angles which were sampled in the simulation. This data is used to represent the BRDF of the modeled surface in a Monte Carlo raytracer which enables synthetic images of these surfaces to be generated.



Figure 2: The measured reflectance functions for the hand made sample, both inked (light) and non-inked (bold). a) -45° incident illumination. b) -75° incident illumination.

The virtual goniospectrophotometer was used by Gondek et al. to generate realistic surface reflectance functions for such materials as pigmented plastic, thin-films, and interference paint. Gondek later enhanced the tool in order to generate reflectance functions for multiple thin films and Christensen filters.

Scallan and Borsch Theory

A theoretical model to compute the reflectance of a sheet of paper was developed by Scallan and Borsch (1972). This theory was based upon optical considerations of the morphology of a paper sheet and was developed because of limitations in the use of the Kubelka-Munk formulas (Kubelka, Munk 31) for the same purpose. The Scallan and Borsch theory is based upon a model of paper that consists of multiple layers of the actual fiber cell walls of the sheet and air. The scattering of light is then accounted for by applying the Fresnel laws to the cell wall/air interface. Absorption is assumed to occur when light travels through cell walls and is governed by the Bouger-Beer laws. In this way, the overall reflectivity and opacity are related to actual physical parameters of the sheet, such as the thickness of the cell walls, the refractive index of the fiber material, and the number of fibers in a cross section of the sheet.

In order to account for the light reflected from a sheet, the sheet is modeled as a series of parallel layers

consisting of cell wall material separated by air. The reflectance and transmittance of a single layer are first calculated by (Scallan and Borsch 72):

$$R_{1} = r + \frac{r(1-r)^{2} f^{2}}{(1-r^{2} f^{2})}$$
(1)

$$T_1 = \frac{(1-r)^2 f}{(1-r^2 f^2)} \tag{2}$$

where

$$f = e^{-2at}$$

 $r = A$ constant corresponding to the
reflectivity of the cell wall/air interface.

- t = Thickness of a layer.
- a = The absorption coefficient of the cell wall material.

Once the values of R_I and T_I for a single layer are determined, the number of layers in sheet (n) is calculated using the basis weight (W), specific volume for the cell wall material (V), and the thickness of the layers (t):

$$n = \frac{VW}{t} \tag{3}$$

Using the value obtained for n along with R_1 and T_1 the sheet reflectance R_0 can be calculated from:

$$R_0 = R_n = R_1 + \frac{R_{n-1}T_1^2}{1 - R_1 R_{n-1}}$$
(4)

The reflectance value R_{∞} , which is important in some opacity determinations, can be calculated by increasing the value of n until R_n becomes independent to further increases in n.

Scallan and Borsch showed these formulations to agree well with empirical measurements (Scallan and Borsch 72, 74). The theory also performed well in situations where the Kubelka-Munk equations had poorer results, such as with very thin sheets and with sheets with lower reflectivity due to high absorption fiber material. The superior performance of the morphological approach is attributed to the fact that it uses constants (f, n, a) that are based upon measurable physical quantities. The specific scattering and absorption coefficients of the Kubelka-Munk equations do not take into account the actual physical construction of the sheet, nor the optical laws that apply to it. It should be noted that the actual measurement of the constants in the morphological model is experimentally tedious, and for this reason the Kubelka-Munk theory still enjoys the most use in the industry.

Laying down the raw paper fibers in multiple layers during manufacture, referred to as *formation*, has a significant impact on the optical properties of the



Figure 3: Reflectance as a function of the number of layers n. Simulation results (diamonds) vs. theoretical predictions (line) of Scallan and Borsch formulations.

material. Non-uniform mass distribution is one result and it is not accomodated by the Kubelka-Munk assumptions. Scallan and Borsch (1974) also addressed this problem. By letting the number of layers (n) and the thickness of the layers (t) vary, they were able to integrate the effects of poor formation into the calculation of opacity and reflectance. They reasoned that the overall reflection of a sheet could be calculated by taking a weighted sum of point reflectances. Each point reflectance was calculated according to Equation 4, considering the number of layers present in the cross section at that specific point. The overall reflectance is then:

$$R_{\overline{n}} = F_1 R_1 + F_2 R_2 + F_3 R_3 + \dots$$
 (5)

where $R_{1}, R_{2}, R_{3},...$ are the reflectances of sheets composed of 1, 2, 3,... layers respectively, and $F_{1}, F_{2}, F_{3},...$ are the fraction of the total area that is composed of these numbers of layers. By assuming that the number of layers across a sheet follows a Poisson distribution, Equation 5 can be re-written as:

$$R_{\overline{n}} = e^{-\overline{n}} \left[R_1 \frac{\overline{n}^1}{1!} + R_2 \frac{\overline{n}^2}{2!} + R_3 \frac{\overline{n}^3}{3!} \dots \right] \quad (6)$$

where \overline{n} is the average number of layers across the entire sheet.

Measurement and Simulation Results

Given that the factors governing the optical characteristics of paper are quite numerous, we would now like to focus our discussion on a few specific optical phenomena that we found to be of interest for our study. We hope to provide some insight into the mechanics of these specific effects by investigating them through the use of both optical measurements and geometric models used in conjunction with our virtual goniophotometer tool. The phenomena of particular interest that were well suited for this type of exploration included:

- •the effect of formation (mass distribution) on overall reflectance;
- anisotropic reflectance functions caused by fiber orientation;
- •retro-reflectance in the incident direction (back-scatter).

The optical effects we chose to focus on were of interest for a number of different reasons. The effects of non-uniform mass distribution have been noted in the literature and theoretical models have been developed to account for it, thus it makes for an interesting case to compare with our simulated models. The second phenomenon, anisotropic reflectance functions, are not directly noted in the literature, but can logically be extrapolated from work that has previously been done regarding the effects of fiber orientation on formation. Accounts of the last phenomenon, back-scatter, were not found anywhere in our literature review, but were noted after the first measurement results were studied. We had no immediate explanation for this surprising result, and thus felt it would be a unique effect to attempt to reproduce in our simulations.

In the process of having optical measurements performed, paper samples from a variety of bond grades and types were gathered with the help of individuals in the paper industry. From these samples, representative specimens were chosen based on such criterion as their similarity to printing papers (those used in the labeling applications for example) and their ability to display the phenomenon we are investigating. The chosen specimens were sent to a third party laboratory for optical profiling. This process involved measuring their reflectance functions under the illumination of a 670 nm laser for a number of different spatial orientations.

In addition to the measurement results, geometric models were carefully constructed in software for use with our virtual goniophotometer. Reflectance functions, similar in form to those of the measurements, were generated by simulations and were compared to both the measurement results and the theoretical formulations discussed in an earlier section. The models and the simulation results provide some insight into the mechanics of the specific optical effects we choose to



Figure 4: Reflectance as a function of the (average) number of layers n. Simulation results for the uniform model (solid) and the Gaussian distributed model (dashed).

concentrate on in this study.

The remainder of this section is divided into four subheadings. First we will overview the paper samples and the general results of the optical measurements performed on them. Next we will detail our exploration of formation effects and compare the results of our models with the established theories which were discussed in a previous section. The third subsection of this section overviews the degree to which anisotropic tendencies were found in our measurements and our reproduction of these effects in simulation. Finally we will discuss the significant back-scattering phenomenon which were found in our measurements. We postulate that this effect may partially be the result of the geometry of specific materials used in the coatings of the samples displaying this effect.

Paper Samples and Measurement Results

A broad range of paper samples were donated by individuals in the paper manufacturing and distribution industries for this study. These samples included sheets constructed with a variety of pulp materials, coatings, and processes. Information regarding the composition and manufacture were provided along with the samples. The two specimens from this lot chosen for measurements were singled out because they were representative of sheets used in applications in which their optical properties are important. The two sheets also provided contrasting cases with the regard to the effects we are investigating.

The first specimen selected was very similar in look and feel to paper used in printing applications such as the labeling of canned goods. This sample was machine manufactured, provided by Unisource Worldwide Inc. OR., and goes by the name of "Mountie Matte." The sample was smooth, white, and somewhat glossy in appearance. The basis weight of the sheet was $60 g/m^2$. It was composed of a bleached ground-wood pulp substrate with a calcium carbonate/clay coating, an industrial standard polystyrene binder, and an alkali sizing. The ratio of coating to substrate weight was not available, nor were the relative amounts of materials used in the coating, as this information is viewed as proprietary by the



Figure 5: Geometry of reflected light from a single oriented fiber. a) Incident light in the machine direction. b) Incident light in the cross-machine direction.



Figure 6: Measured reflectance of the Mountie Matte sample for 75 degree incident illumination. Curves shown represent illumination parallel to both the cross (dashed) and machine (solid) directions.

manufacturer. Normally coatings account for 10 to 15% of the overall basis weight of a sheet (Bureau 89). Since this sample was machine manufactured, it was one of the candidates we predicted would display anisotropic tendencies in its reflectance functions, and it was therefore an appropriate choice for our investigation of this effect. This sample also showed a significant amount of retroscatter, which was also of interest to us.

The second specimen was typical of writing grade bond or paper that is used for important printed documents. This sample was more roughly textured than the previous, slightly yellow in hue, and diffuse in appearance. The sheet was composed of 97% unbleached softwood pulp (Jack Pine and Red Pine), 3% hardwood pulp (Minnesota Aspen), and contained no fillers, coatings, or sizings. This sheet provided an excellent contrast to the previous specimen because it was hand formed by Potlatch Industries Cloquet MN. and did not contain any secondary materials. Because the sheet was not manufactured by machine, we expected to see no anisotropic tendencies in its reflectance functions. This specimen also benefited from the fact that it was composed only of pulp and thus was easier to accurately model than the previous specimen. Since this sheet was uncoated, it would also provide us with an opportunity to confirm our speculation regarding the role of coatings in the back-scatter phenomenon which was discovered in the measurement of the first sample.

After the samples had been chosen, we devised a measurement scheme which would provide a broad overview their reflective properties. In order to also observe in what way printing augmented the reflective properties of the sheets, two specimens from each sample were produced: one inked and one plain. The inked specimens were generated by printing a large black region onto the samples with typical office-type computer printers. Tests were conducted on the Mountie Matte sample according TAPPI specification T 409 in order to determine its machine direction. Measurements were taken both along and against this direction in order to



Figure 7: Reflectance functions for the fully oriented paper web model. Curves shown represent illumination parallel to both the cross (dashed) and machine (solid) directions.

record any anisotropic tendencies in this specimen's reflectance functions. Perpendicular sets of measurements were also taken for the hand formed specimen for comparison, although no attempt was made to determine its "machine direction." Measurements from all four specimens were taken for a number of incident illumination angles in order to record any effects in the reflectance functions which were related to spatial orientation.

The optical profiling of the specimens were performed by a third party laboratory, and involved measuring the light scattered from the samples when illuminated by a laser. A 670 nm laser in an (S)-polarization state was arranged so that the incident beam was at a specified angle with the normal of the specimen. Measurements were then taken by recording the magnitude of the reflected light at discrete locations on the hemi-circle perpendicular to the horizontal plane of the specimen that is formed by sweeping out the angles from -90° to $+80^{\circ}$ with respect to the normal. This type of measurement sweep was performed for a family of incident angles on each specimen. A similiar experimental method was employed in the Letteri et. al. (1991) study of light scattering from glossy coated papers. Measurements were also taken with the specimens rotated 90° in the horizontal plane, in order to record any anisotropies in their reflectance functions. These latter measurements were only performed on the non-inked samples.

Reflectance functions for a family of incident angles on the first specimen is shown in Figure 1. The most prominent characteristic of these functions is the marked peak located around the mirror reflection direction. This effect was expected since the sample was somewhat glossy in appearance. The inked samples display a lower overall reflectance in all cases, which was also expected, however there are some anomalies present at glancing exit angles for these cases. The reflectance of the inked samples do not seem to drop off as rapidly as the non-inked samples at these extreme exit angles. In all cases except for 75° incident, the reflectance curves for the inked samples crosses over those of the non-inked at an exit angle of approximately -72°.

We can only speculate as to the cause of this anomaly, however it may be related to an increase in surface reflection caused by the introduction of the ink. Another unexpected result was the significant back-scatter, or reflectance around the angle of incidence. There is a noticeable rise in the reflectance curves around the retroreflection angle in all cases for the non-inked specimen, and, for 20° incident (curve b), the back-scatter is actually of the same magnitude as the specular reflection. (The discontinuities in the curves at the edges of these back scatter "peaks" are an artifact of the laboratory's We found this feature in the experimental set up.) reflectance functions surprising since such phenomenon was not mentioned in our literature survey. It was only after detecting this feature in the initial measurements that we decided to focus on this issue with further measurements.

The measurement for the hand made samples are shown in Figure 2. These reflectance functions display a more diffuse response than the previous case. The presence of back-scatter is also much less prominent in these curves. The curious aspect in these functions is the relative parity in the magnitude of the specular peak for both the inked and non-inked samples in their -75° incident measurements. Again this result is puzzling and may be due to Fresnel effects which dominate the reflection at these glancing angles.

Modeling the Effects of Formation on Reflectance

Recall from our discussion in a previous section that formation is a measure of the degree of non-uniformity in a sheet's distribution of mass. Because of the random nature in the construction of the fiber web, all sheets will exhibit some degree of non-uniformity in this distribution. The optical consequences of formation are readily seen by viewing transmitted light through a typical sheet. The blotchy appearance of the light is due to the point-to-point variance in mass across the sheet. Since there is a non-linear relationship between mass, or basis weight, and reflectance/opacity, the effects of formation complicate the application of theoretical treatments to real sheets. Sub-regions of a sheet that contain below average mass have lower reflectance values than what is compensated for by the regions of above average mass. This results in a discrepancy between values predicted by the theories and reflectances measured for real sheets, because the mass is not uniformly distributed in the latter case. Corrections to the theories which accounted for these discrepancies were developed by various researchers, as was reviewed in a previous section. By paralleling the development of one of these modified theories as our first investigation into the modeling paper, we will show that our simulations can produce reflectance functions that agree well with theoretical results.

For our initial attempts at modeling a sheet, we choose to duplicate the morphological model developed by Scallan and Borsch. A sheet is represented in this model

as a series of alternating layers which correspond to the bonded cell walls of the fibers and the air spaces of their cores/interstices. This model provided an excellent introductory case for our investigation since its geometry is relatively easy to reproduce and because we could directly confirm the resulting reflectance functions with the theoretical predictions of the Scallan and Borsch formulations. We were also able to parallel in our simulations the extensions made by Scallan and Borsch to account for non-uniform mass distribution.

The original Scallan and Borsch formulations for the reflectance of a sheet were derived by using established optical relationships such as the Fresnel equations and the Bouger-Beer laws to determine the predicted reflectance from the multiple layers of their model. These formulations are discussed in earlier in an earlier section and are listed as Equations 1-4. The foundations of the virtual goniophotometer are the same established optical relationships used by Scallan and Borsch to develop their theory, thus it is reasonable to directly compare the results from each source. The primary parameters of the Scallan and Borsch formulations include: the number of layers in the sheet, the thickness of these layers, and the index of refraction of the cell wall material. By using the same



Figure 8: Reflectance functions for the randomly oriented paper web models. a) Web modeled with 15 degree mean variance from machine direction. b) Web modeled with totally random angular distribution. (no machine direction) Curves shown represent illumination parallel to both the cross (dashed) and machine (solid) directions



Figure 9: Reflectance function for the Calcium Carbonate model.

values in the construction of our model, we were able to generate almost identical reflectance functions to those predicted by their theory.

Our model consisted of uniform alternating layers of cell wall material and air. The cell walls had an index of refraction of 1.56, and the thickness of these layers was set to 5 mm, which corresponds to values reported by Scallan and Borsch for typical pulp mixtures. A series of simulations were run on subsequent models, each with an increasing number of layers, and the results are shown plotted against theoretical calculations for identical parameters in Figure 3. As we can see the two curves show excellent agreement.

In order to account for the effects of formation on the reflectance of a sheet, Scallan and Borsch developed a separate set of equations from their original formulations. While they did not explicitly specify a new geometric model in connection with these corrections, a model was easily derived from their new formulas for our study. The primary difference in their new approach was that the sheet was no longer viewed as having a uniform distribution of mass, and the overall reflectance was now calculated by integrating the local area reflectances over the area of the sheet. This variation in mass manifests itself in the nparameter of the original equations, so the new equations were derived by allowing n to vary. Using a Poisson distribution function for this variance, Equation 6 was derived, as discussed previously.

We were able to make an analogous modification to our simulated model. The model was broken up into discrete areas and the number of layers in each area was allowed to vary in a similar fashion as the n parameter of the theoretical formulations. The new model was constructed by an array of 50 by 50 mm rectangular areas with the number of layers in each area randomly assigned according to a Gaussian distribution function. All the other parameters of this model were identical to those of the preceding one. A series of simulations were run using this model with subsequently increasing numbers of (average) layers. The reflectance functions generated for these non-uniform sheets were consistently lower than their uniform counterparts as shown in Figure 4. Similar results are also predicted by the augmented Scallan and Borsch theory.

Anisotropic Reflection Functions due to Fiber Orientation

A high degree of correlation between the simulation and theoretical reflectance functions was shown in the preceding examples, however, this investigation did not exploit the virtual goniophotometer's ability to distinguish spatial detail in the reflectance functions. The theories and simulations that dealt with formation effects only considered the overall diffuse reflectance of a sheet and did not cover any of the spatial aspects of the scattered light. One effect that would manifest itself spatially, and thus would not be evident from inquiries like the preceding one, is the effect that the orientational preferences exhibited by fibers would have on a sheet's reflectance functions. Anisotropies in the mechanical properties of a sheet have been associated with such phenomenon, and would be expected to be similarly present in the optical characteristics of a sheet. The optical effects of such phenomenon have not been documented in any of the literature we reviewed; in fact, there was not much focus on any spatial aspects of scattered light from paper in this previous research.

Fibers that are mechanically deposited on the wire during formation of the base stock will tend to align themselves parallel to the direction the wire is moving. The direction of this orientation is called the grain or machine direction, and its effects on various mechanical properties are well documented. In addition to the mechanical properties that this angular preference influences, studies have also linked this phenomenon to the degree of non-uniformity in the distribution of mass, or the formation index of sheets. However, no research has been done regarding the optical consequences this phenomenon might have. Despite these oversights, there is still reason to believe that fiber orientation introduces anisotropy in the optical properties of sheet similar to those encountered in the mechanical properties. Because of the geometry of the fibers in the web, incident light that is parallel to the machine direction should reflect light in a more specular fashion than identical light incident in the cross direction. Figure 5 shows these situations for a single fiber oriented in the machine direction. This phenomenon would manifest itself only in a full spatial profile of the reflected light.

Prior to sending it to the laboratory for measurements, tests were conducted on the Mountie Matte paper sample in order to determine its machine and crossmachine directions. These tests were conducted according TAPPI test specification T 409. to Separate measurements were taken for incident illumination parallel to both these directions. The resulting reflectance functions show a greater specular peak for light incident along the machine direction than for identical illumination in the cross direction. This confirmed that anisotropy exists in the reflectance functions at least for this sheet. Figure 6 clearly shows this effect for 75° incident illumination. The effect was less pronounced for other



Figure 10: The effects of simulated calcium carbonate. a) Reflectance functions for the calcium carbonate over simulated web model. b) Similar web without coating.

incident illumination angles. Given that this paper was machine manufactured, we were surprised to find that this anisotropy was not as dramatic as we had expected.

The anisotropic tendencies of this sample may have been subdued because of its coating. The geometry of the coating particles for this sample are relatively isotropic in the horizontal plane of the sheet, and thus would not contribute to any anisotropic effect in the reflectance function. The presence of these particles in the top surfaces of the sheet may somewhat mask the anisotropy inherent in the web substrate below. Similar measurements conducted on uncoated. machine manufactured paper could confirm this speculation, but was not attempted in this study. Regardless of the degree to which it is present, there is clearly some anisotropy to the reflectance functions of this sample, and exhibited by Figure 6.

In order to reproduce this anisotropic effect in our simulations, we had to abandon the layer model used in the previous exploration, and model the actual fiber web. The base stock web of a sheet is composed of a collection of slender transparent fibers which resemble hollow cylinders, and are oriented somewhat randomly in the horizontal plane of the sheet. We choose to use concentric cylinders to represent a fiber for our model. In the pulping and web formation process, fibers may undergo deformations and come to resemble thin ribbons. Studies have shown that fibers will generally deform in predictable ways according to such factors as the fiber source, pulping technique, and method of construction (Jayme and Hunger 62, Wrist 67). For the purposed of our study, we choose a to model a simple deformation that compressed (scaled) the fibers in one direction perpendicular to their long axis. We set the dimensions of our cylinders to be 2 mm long, .02 mm in diameter for the outer cylinder, and 0.01 mm diameter for the inner cylinder. These numbers correspond to the dimensions of typical wood pulp fibers (Dodsen and Fekih 91).

Our first model of the web was a "fully oriented" sheet. 1000 of the deformed cylinders were placed side by side, oriented parallel to the machine axis to form one layer of fibers for our modeled sheet. Ten of these layers were then stacked atop each other to complete the model. The reflectance functions generated by this model are shown in Figure 7. As we can see from this figure, there is clearly a difference in the reflectance functions corresponding to the grain direction. The characteristics of these curves are similar in nature to the measurement results which were shown in Figure 6.

This fully oriented model is sufficient for illustrative purposes, however, no real sheet would have such a uniform configuration. In order to create a better representation of the web, the angular orientation of the cylinders that make up the model must be allowed to vary with a certain degree of randomness. By introducing randomness, we are able to model the fiber orientation more accurately; however, the optimal packing arrangement of these cylinders is no longer a trivial matter. The packing of geometric primitives is known to be computationally difficult (Gardner 68, Boyd 73). For the purposes of constructing our model, we relaxed one of the other parameters in order to facilitate control over the orientation. To maintain the statistical accuracy of the fiber orientation that we required for the study, we allowed the Z dimension of the model to vary such that the web was allowed to grow unconstrained along this direction during its construction. This translates into a discrepancy between the overall thickness, or caliper, of our modeled sheet and what would be found in real sheets with equivalent amounts of fiber mass. We felt this simplification was justified since we were interested in the optical effects of strictly fiber orientation and not the density of their packing.

Using the same number of fibers as the fully oriented sheet, a random web was constructed to have a mean angular variance of 15° from the machine direction. This variance corresponds to a highly oriented web, but is more realistic than the previous model. The algorithm used to construct this web attempted to pack the cylinders as densely in the horizontal plane as possible in order to produce adequate coverage across the area of the sheet. However, during the construction we allowed subsequent cylinders to "slide down" the Z axis so that they did not intersect with cylinders already in the web. This algorithm did not produce a particularly dense fiber packing, but did maintain the angular distribution of the fibers.



Figure 11: Comparison of measurement results for coated and uncoated paper samples. a) Mountie Matte specimen, coated. *b*) Handformed sample, uncoated. Curves shown represent inked (light) and non-inked (bold) specimens.

The reflectance functions produced by the simulations on this model are shown in Figure 8a). There is still a significant anisotropy exhibited in the reflections functions of this model. While the fibers of this model were highly oriented along the machine direction, the overall geometry is more similar to real sheets than was the case for the fully oriented model. Figure 8b) shows the results of simulations on a similar model that had *no* orientational preferences. From the results we can see that the anisotropic effects disappear for a web that has fibers oriented in an entirely random fashion.

Simulation of Retro-Reflectance

Like the anisotropic effect studied in the previous section, we did not encounter any references to the phenomenon of retro-reflectance from paper in our literature survey. However, while we expected to find anisotropy in the measurements, we were somewhat unprepared for the significant amount of retro-reflection exhibited in these same results. The origins of this backscatter effect are somewhat puzzling, however, we theorize that it is partially a consequence of the geometry of the coating used in this sample.

One of the most notable characteristics found in the measurement results of the Mountie Matte specimen was the significant amount of retro-reflection it displayed over all incident angles. In examining the curves shown in Figure 1, we notice a distinct local peak in the reflectance around the back-scatter angle in all cases. (For 0° incident, the mirror and back-scatter angle are identical.) None of the geometries we had modeled up to this point had produced reflectance functions with features that in any way resembled this retro-reflection peak. One aspect missing in these models, but present in the Mountie Matte specimen, was a coating.

The aspect of a coating that has the most influence on the spatial distribution of scattered light is the shape of the particles used in the coating. We concentrated on this aspect of the coating since retro-reflectance is a spatial phenomenon. Clay and calcium carbonate are the two primary components of the coating mixture used in the Mountie Matte specimen. The platelet-like structure of clay tends to enhance the specular reflection of the scattered light. Calcium carbonate is more spheroidally shaped, and thus tends to scatter light in all directions. The spherical geometry of calcium carbonate may also be contributing to the observed back-scatter in the reflectance functions of the Mountie Matte specimen. This line of reasoning was further explored by the construction a model of calcium carbonate which was used in the virtual goniophotometer to generate back-scatter reflectance functions.

A model of Calcium Carbonate was constructed by creating a volume filled with small transparent spheres. These spheres had a radius of 500 um, and their index of refraction was set to be 1.6 corresponding to values reported in (Johns, 77). The spheres were packed in a volume which measured 250 x 250 x 170 mm deep by a simple random insertion algorithm. This method is identical to the one employed by Gondek et al. (1994) to model pigmented plastics. Since the geometry of spheres will produce relatively diffuse scattering, significant amounts of energy in the reflectance functions generated by this model were in the retro-reflectance direction. Figure 9 shows the peak of this function to be in the retro-reflection direction; it is not a specular peak. The dominance of the back-scattering effect in the reflectance functions of this model are not surprising given the nature of its geometry; but these functions are not very suggestive of the ones found in the measurement results.

A second model was developed in which the calcium carbonate coating was placed on top of a simulated base stock web. Since real coatings have a binder applied to them in order to secure them to the substrate, the spheres in this new model were partially embedded in a medium with an index of refraction of 1.1 which represented the binder. Simulations were re-run on this second model, and the results are shown in Figure 10. These reflectance functions show similar characteristics to those of the measured sample. There is both a specular peak, and a retro-reflectance peak present in the curve. Simulations run on a similar web without the coating reveal that this effect is not present in the absence of the coatings as shown in b) of Figure 10. The measurement results on the handmade sample also support this claim. These results are compared with the Mountie Matte in Figure 11

and it is clear that the uncoated sample has much less significant back-scatter than the coated sample.

While these results support the theory that the geometry of the calcium carbonate contributes to the observed retro-reflectance, it must be noted that the models used in these simulations represented a simplified version of the geometry of calcium carbonate. There is reason to believe that the geometry of a pigment plays some role in the observed back-scatter phenomenon. However, with only these preliminary results, it would be premature to conclude that this is the primary source of this effect.

Conclusions

In this paper we have used computer graphic reflection modeling techniques to study the surface reflection of paper. We began by having the reflectance of several paper samples measured. These measurements were made across the hemisphere above the sample at several different angles of incidence. Next we employed a virtual goniospectrophotometer to build and measure several different models of paper. In this work we attempted to duplicate the effect of formation, fiber orientation, and surface coating on the reflectance of the paper. We found good correlation between our simulations and both the analytical and measured reflectances of our paper samples.

This work is a first step along a path towards the construction of a complete appearance modeling and evaluation system. The use of computer graphic light modeling and rendering techniques should make it possible to model the structure of a hypothetical material, simulate the reflectance from this surface, and create a picture of what an object coated with this substance will look like. In this way an appearance engineer or an imaging scientist will have a computer aided design tool that can be used to explore new surface coatings and printing techniques without having to invest the time, effort, and money necessary to create a prototype of the new paint, plastic, or inked paper that is under development.

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